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FORMATIVE ASSESSMENT IN MATHEMATICS: A THEORETICAL MODEL

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Abstract. This paper concerns the description and discussion of a European research program about the examination of formative assessment in the teaching and learning of mathematics. The focus is on the presentation of a questionnaire constructed for the purposes of the research program, for tracing the mathematics teachers' beliefs regarding the use of formative assessment in mathematics teaching and learning. The development of the questionnaire was based on an extensive literature review in mathematics education in relation to formative assessment and the teachers' educational beliefs. A part of the results of the pilot administration of the questionnaire are included and also expected results of the study are presented, regarding the structural organization of formative assessment in mathematics and the different dimensions that are related to formative assessment.

Key words: formative assessment, mathematics, teachers' beliefs questionnaire, feedback

1 INTRODUCTION

This contribution is about the description and discussion of an ongoing research program about Formative Assessment in mathematics. The project is entitled *Formative assessment in mathematics for teaching and learning (FAMT&L)*[1]. In this paper the focus will be mainly in the first part of the project, which consists of the study of the mathematics teachers' beliefs about the use and the role of formative assessment in mathematics. In fact, the emphasis is given on the description of a designed questionnaire for tracing the teachers' beliefs, which takes into account a spectrum of dimensions related to formative assessment in mathematics. Based on these dimensions, we propose a structural organization for describing the construction of the teachers' beliefs about formative assessment in mathematics.

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In the next parts of the paper a more detailed description regarding our project will be provided, about our aims, objectives and methodology. Then, a synthesis of the literature review will be presented regarding the role of beliefs in the teaching and learning of mathematics and the role of formative assessment in mathematics. Further on, the procedure for developing our research instrument and a more detailed description of the relevant questionnaire will be provided. Some first results from our pilot study and the proposed structural organization of formative assessment will be presented and explained, in relation to the main axes of the research.

2 THE FAMT&L PROJECT

The FAMT&L project proposes an innovative path that, starting from an investigation of the mathematics teachers' beliefs about formative assessment, will get to design a virtual environment (a web repository) for in-service teachers' training. This learning environment should provide a variety of tools and objects (examples of learning contexts, video of situations of teaching mathematics, assessment tools, training paths and their specific use in the teaching of mathematics), including a guideline to be used in in-service secondary schools teachers training courses.

There are five EU partners in the project: The Alma Mater Studiorum Università di Bologna – Departments of Education and Mathematics, which is the Project Coordinator, the University of Cyprus – Department of Education, the University of Applied Sciences and Arts of Southern Switzerland – Department of Formation and Learning, the Cergy-Pontoise University – University Institute of Teachers Training and the Inholland University of Applied Sciences.

The main objectives of this project consist in realizing a survey on the mathematics teachers' beliefs and practices concerning assessment in classroom, in designing and implementing a web repository for the mathematics teachers training about the proper use of formative assessment in teaching-learning situations and in elaborating a training model (or methodology) for mathematics teachers training in secondary school. This training methodology should improve teachers' skills regarding the use of formative assessment in mathematics education in order to promote effective learning for all students.

The first part of the project is dedicated to the analysis of teachers' learning beliefs and needs regarding formative assessment, through specific qualitative and quantitative research methods (observations, interviews, questionnaires, survey, etc.). The collected data will be used for designing an effective training model for the teachers of mathematics. The design and development of this training model will be realized as an action-training research, where teachers will be actively involved and trained to develop mathematics teaching and assessing competences as well as transversal competences such as reflexive practice, self-assessment, planning and reporting methods, professional empowerment. All the products will be included in the web repository in order to give to trainees as much stimulus and tools as possible.

In this paper we are going to describe and discuss the study of mathematics teachers' beliefs for formative assessment. For this purpose a questionnaire was developed, for collecting the teachers' beliefs regarding various dimensions of formative assessment. The procedure for developing our research tool is described in the next session.

2.1 THE SURVEY OF THE TEACHERS' BELIEFS FOR FORMATIVE ASSESSMENT

In relation to the purpose of our research, an extensive study of literature in the fields of beliefs and assessment in mathematics education was done. The results of this literature review were used for determining our axes of investigation and for constructing a questionnaire for examining the teachers' beliefs about different dimensions of formative assessment. Finally we concluded to four main research axes; the teachers' beliefs about 1) the purpose, 2) the techniques, 3) the results of formative assessment in mathematics and 4) the teachers' training in formative assessment in mathematics. Therefore, our research questions are the following:

1. What are the mathematics teachers' beliefs about the purpose of formative assessment?
2. What are the mathematics teachers' beliefs about the techniques that should be used of formative assessment?
3. What are the mathematics teachers' beliefs about the way the results of formative assessment should be used?
4. What are the mathematics teachers' beliefs about their training in using formative assessment?

Based on the literature, the various authors' opinions and research results were transformed to statements to be included in our questionnaire. Previous relevant research instruments were also traced, parts of which were taken as examples for forming some of our statements. The different statements were then grouped according to our preliminary research axes. After coming to a complete set of research axes, including a large number of questions in each axis, the questionnaire was sent to all the partners of the project for content validation. Next, some axes were merged and some questions were eliminated, in order to reduce the extent of the questionnaire. The corrected version of the questionnaire was sent again to all the partners for revision. After the finalization of the questionnaire, each partner was responsible for its translation in the relevant language and its administration.

The questionnaire will be administered to lower secondary schools mathematics teachers.

The questionnaire is comprised of six parts. In the first part (Part A) the participants' demographics are asked. This part includes questions mainly about the participants' gender, age, education and teaching experience. In each of the rest five parts the participants mostly have to express their agreement or disagreement to different statements on a 4-point Likert scale (1=strongly disagree, 4=strongly agree). Negative statements are used as well, in order to increase the validity of the questionnaire. Specifically, Part B includes 10 statements examining the first research axis, which is about the purpose of formative assessment. In Part C there are 21 statements about the use of different formative assessment techniques. In the next part (Part D) 7 statements are found, examining the participants' beliefs regarding the use of the results of formative assessment. Part E comprises of 12 statements for tracing the teachers' beliefs concerning mathematical errors. The last part (Part F) includes 16 issues of assessment on which the teachers' would like or not to be further trained on.

A pilot administration of the questionnaire was performed by all partner countries. In the next sessions some results of the pilot administration of the questionnaire in Cyprus will be discussed. The participants were 21 secondary school mathematics teachers, aged between 30 and 60 years old, form both genders and having teaching experience from 5 to 40 years.

The following sessions are organized in a way to make the relation between our literature review, our main research axes and the construction of our questionnaire more explicit. Each session corresponds to a research axis and includes some of the basic theoretical elements we used for designing our questionnaire and some representative statements with the corresponding first results.

3 PURPOSE OF FORMATIVE ASSESSMENT

The National Council of Teachers of Mathematics Assessment Standard (NCTM, 1995) define assessment as "the process of gathering evidence about a student's knowledge of, ability to use, and disposition towards mathematics and of making inferences from that evidence for a variety of purposes" (p.3). In accordance to this, Harlen (2000) points out that "children have a role in assessment for this purpose since it is, after all, the children who do the learning" (p.112). That is why many researchers stress that assessment must be formed "for" learning and not "of" learning, as it is generally acknowledged that increased use of formative assessment (or assessment for learning) leads to higher quality learning (Wiliam, Lee, Harrison & Black, 2004).

In this sense, Nicol and Macfarlane-Dick (2004) argued that formative assessment should be an integral part of teaching and learning in higher education. We agree with this opinion, emphasizing that the use of formative assessment in teaching can have many benefits on one hand on improving the students' mathematical learning but also the development of positive beliefs towards the learning of mathematics, and on the other hand in helping the teachers in doing proper adjustments according to their students' needs. Our opinion is also in line with other researchers' definitions that stress the effects of formative assessment in modifying learning in relation to the students' needs. Van De Walle, Karp and Bay-Williams (2013) define formative assessment as "an along the way evaluation that monitors who is learning and who is not and helps teachers to form the next lesson". Wiliam (2007) claims also that "to be formative, assessment must include a recipe for future action" (p.41). FA then is a strategic process which uses evidence regarding the extent of student knowledge (declarative knowledge) and skill (procedural knowledge) to support further learning (Clark, 2011a) and as such increases student motivation and engagement (Cauley & McMillan, 2010).

Additionally to the aforementioned focus points about the monitoring of teaching and learning, the role of feedback is also emphasized in many other definitions about formative assessment. According to such definitions, formative assessment refers to assessment that is specifically intended to provide feedback on performance for improving and accelerating learning (Sadler, 1998). Cauley and McMillan (2010) add to this by defining formative assessment as a process through which assessment elicited evidence of students' learning is gathered and instruction is modified in response to feedback. In the same sense, for Nicol and Macfarlane-Dick (2004) formative assessment, besides providing a framework for sharing educational objectives with students and for charting their progress, it can also generate feedback that can be used by students to enhance learning and achievement and by teachers for adjusting their teaching practices in order to correspond to their students' needs. It is thus obvious that formative assessment can have a powerful influence on achievement by providing meaningful feedback to students as to what they know and where they make errors or have misconceptions (Hattie, 2009).

Summarizing, a definition combining all the points stressed previously is the one provided by Popham (2008, p.5), who characterize formative assessment as "a process used by teachers and students during instruction that provides feedback to adjust ongoing teaching and learning to improve students' achievement of intended instructional outcomes." This definition is accepted by the Formative Assessment for Teachers and Students (FAST) group as the most accessible to educators (Clark, 2011b).

The teachers' beliefs about the purpose of formative assessment were of high interest for the purpose of our research, as the way the teachers' define formative assessment is related to the techniques they adopt and the way they interpret and use their results. Thus, using the aforementioned definitions which express the purpose of formative assessment, we formed a particular part of our questionnaire for answering our first research question. This part included statements examining the teachers' beliefs about the role of formative assessment and the benefits of its use in the teaching and learning of mathematics. Table 1 presents representative statements and some of the first results regarding this part of the questionnaire.

These first results show that the teachers mostly agree with these statements, indicating that they recognize the contribution of formative assessment for the identification of what their students are learning and what their students' strengths and weaknesses are. The teachers believe also that formative assessment can be used for defining the validity of their teaching work. We also observe that the majority of the teachers agree that formative assessment is closely related to the understanding of mathematics and to the ability of analysis and synthesis. On the other hand, the statement which relates a different purpose of assessment in mathematics, that is memorization, is less supported by the teachers. These observations show that these teachers have a more conceptual idea about the teaching of mathematics, which does not only focus on the acquisition of knowledge, but also on the development of mathematical abilities.

Table 1

Frequencies, mean and standard deviation of teachers' beliefs about the purpose of formative assessment

	No answer	**Strongly Disagree**	**Rather Disagree**	**Rather Agree**	**Strongly Agree**	**Mean**	**SD**
Formative assessment establishes what students have learned in mathematics.	0	1	1	17	2	0,74	0,15
Formative assessment identifies the students' strong and weak abilities in mathematics.	0	0	0	18	3	0,79	0,09
Formative assessment identifies how students think in mathematics.	0	0	2	16	3	0,76	0,12
Formative assessment should assess the students' ability to apply mathematics in unfamiliar everyday situations.	1	0	2	12	6	0,76	0,23
The different assessment methods aim to assess the students': Knowledge (memorization): the ability to memorize rules, axioms, theorems and other mathematical information	0	0	5	13	3	0,73	0,16

Comprehension (understanding): the ability to perceive mathematical meaning and to transform mathematical ideas from one form to another	0	0	0	15	6	0,82	0,12
Analysis: the ability to analyze information and to arrive to mathematical conclusions	0	0	0	15	6	0,82	0,12
Synthesis: the ability to organize mathematical ideas altogether to form a complete image that has meaning	0	0	0	16	5	0,81	0,11
Assessing my students' is very useful for me, because it gives me a chance to verify the validity of my work.	0	0	1	14	6	0,79	0,12

4 TECHNIQUES OF FORMATIVE ASSESSMENT

The second focus point for our examination is the teachers' beliefs about the use of particular techniques and practices for implementing formative assessment and about factors that influence their choice of particular techniques and practices. In fact, assessment practices and their outcomes on the students' learning, but also their affective domain has drawn the interest of different researchers in the last 30 years (i.e Crooks, 1988; Black & Wiliam, 1998). Previous works suggest different formative assessment techniques, most of which appear to have common points. For example, Cauley and McMillan (2010) try to highlight some formative assessment techniques by comparing formative with summative assessment. They actually say that the results of summative assessment provide evidence only about the current achievement of the students, at the time the assessment is done. On the contrary, despite the fact that a summative technique can be used in formative assessment, such as a test, the results of FA can provide teachers information about students' misunderstandings and use these information during their teaching in order to provide feedback to students and help them correct their errors. Therefore, Cauley and McMillan (2010) provide particular techniques that should be used in teaching for the effective integration of formative assessment in instruction. Specifically, informal observations and oral questions posed to students while content is being taught or reviewed is a practice that allows ongoing formative assessment. And if the information from the observations and questions to students is accurate, the teacher identifies instructional adjustments that can help improve the students' learning.

From our experiences, observations and oral questions are included in the teachers' repertoire of formative assessment techniques and are very commonly used in teaching. Despite the frequency of their use, we are not sure that these techniques can be included among the most effective ones for formative assessment. However, research indicates that teachers' opinions and preferences are not totally in line with our opinion**.** In fact, Kyriakides and Campbell (1999) examined primary teachers' opinions about the appropriateness of particular techniques of assessment in mathematics. Performance test and structured observation were considered to be the most appropriate methods. On the other hand, unstructured observation and oral question-and-answer were seen to be the least appropriate techniques. Teachers were also asked to express the degree of difficulty of these techniques. The results indicated that unstructured observation was considered to be the easiest technique and oral question-and-answer as the next most easy. In an effort to shed some light to this contradiction, statements examining the teachers' beliefs about the appropriateness of the use of the aforementioned assessment practices were included in our questionnaire (Table 2).

Furthermore, Cauley and McMillan (2010) stress also the power of using the practice of providing clear learning targets to the students. They explain that formative assessment is more effective when students have a clear idea about their teachers' expectations of them, because providing clear expectations enables students to set realistic and attainable goals. Thus, teachers can improve the clarity of student learning targets by providing examples of both weak and stellar work. Furthermore, such knowledge is powerful because students have a good understanding of what they are doing and why the teacher provides them feedback and these help them understand what they are learning, to set goals, and to self-assess. Although this technique was suggested by Cauley and McMillan (2010) for formative assessment in general, we consider that this practice is also important for the formative assessment in mathematics also, as the students' knowledge of their teachers' criteria allows them have a clear idea about the mathematical content they learning or the mathematical processes, the strategies they need to develop and the way they are expected to be involved in the teaching and learning process.

The aforementioned techniques discussed by Cauley and McMillan (2010) and Kyriakides and Campbell (1999) are also found in the "Teaching Quality Papers" published by the General Teaching Council for England (2011). In fact, the General Teaching Council for England (GTCE) considers the effective use of questioning techniques, the use of marking and feedback strategies, the sharing of learning goals to students and peer and self-assessment by pupils to be key characteristics of formative assessment. The GTCE provide also a number of techniques which embody these key characteristics. Some of these techniques include sharing learning goals with pupils, helping pupils know and recognize the standards to aim for, providing feedback that helps pupils to identify how to improve and pupils learning self-assessment techniques to discover areas they need to improve.

Clark (2010) provides a richer list of sixteen formative assessment teaching techniques, suggesting that these techniques engage students in reflective thinking and problem solving. Among these sixteen techniques, higher order questioning techniques, feedback for students as comments and not grades, oral feedback to students, sharing assessment criteria with students, peer assessment and collaborative goal setting with and by students are included. It is obvious that the techniques suggested by Clark are also found in the previous suggestions that were discussed. Therefore, we can see that there is a general agreement between the different researchers in the techniques they consider as important for the effective implementation of formative assessment in the mathematics classroom.

Table 2

Frequencies, mean and standard deviation of teachers' beliefs about techniques of formative assessment

	No Answer	**Strongly Disagree**	**Rather Disagree**	**Rather Agree**	**Strongly Agree**	**MEAN**	**SD**
Formative assessment in mathematics is conducted primarily through informal observations.	2	0	9	10	0	0,57	0,23
Formative assessment is conducted primarily through oral questions posed to students while the mathematical content is being taught or reviewed.	2	0	5	13	1	0,63	0,25
To what degree do you agree that the following assessment techniques are appropriate to be used in the teaching of mathematics?							
a) Unstructured observation	2	0	10	6	3	0,60	0,27
b) Oral question-and-answer	2	0	2	14	3	0,69	0,26
c) Structured observation	3	0	1	10	7	0,71	0,33
d) Interview	2	1	5	11	2	0,62	0,27
e) Performance test for each pupil	2	0	3	12	4	0,69	0,27
f) Multiple choice and	2	0	2	14	3	0,69	0,26
g) Matching questions	2	0	2	14	3	0,69	0,26
h) Sentence Completion	2	2	7	8	2	0,57	0,28

Some characteristics of assessment are embodied in a number of processes like:							
a) sharing learning mathematical goals with students	0	0	0	15	6	0,82	0,12
b) providing feedback that helps students to identify how to improve in mathematics	0	0	0	11	10	0,87	0,13
c) both the teacher and the students reviewing and reflecting on their performance and progress	0	0	2	13	6	0,80	0,15
d) students learning self-assessment techniques to discover mathematical abilities they need to further work on.	0	0	1	13	7	0,82	0,14
Formative assessment is most effective when students have a clear idea of what the teachers expect of them.	0	0	1	14	6	0,81	0,13
Teachers can improve the clarity of student learning targets by providing examples of both weak and stellar mathematical work.	1	0	1	17	2	0,73	0,19
Providing clear expectations enables students to set realistic, attainable goals.	0	0	1	14	6	0,81	0,13

However, our first results indicate that the teachers do not always share these opinions about the appropriateness of particular techniques (Table 2). Our first results show that the teachers mostly agree with the effectiveness of sharing their targets with their students and recognize the benefits of this technique on the students' learning and the development of students' skills, like setting goals and self-assessment skills. Regarding observations (informal, unstructured), the teachers have a moderate opinion regarding their use and appropriateness. This is also the case for using sentence completion as a formative assessment technique. On the other hand, this is not the case for the use of structured observation, which the teachers rank as the most appropriate technique. Furthermore, teachers also find appropriate the use of oral questions, of performance test for each pupil, of multiple choice and matching questions. Our first results are close enough to the results of Kyriakides and Campbell (1999), but of course we do not attempt to generalize these results due to the very limited sample size.

Besides the examination of the teachers' beliefs about the use of particular techniques, we considered important to extract factors that influence the teachers' preferences towards specific techniques. Therefore, our questionnaire included statements asking the teachers opinion about factors that affect their ability to apply different assessment techniques and about how skilled they think they are in applying them.

Table 3

Frequencies, mean and standard deviation of teachers' beliefs about factors determining their choice of techniques of formative assessment

	No Answer	**Never**	**Rarely**	**Often**	**Always**	**MEAN**	**SD**
How often do the following factors affect your ability to apply different assessment methods?							
a) The curriculum workload	0	0	3	13	5	0,77	0,16
b) The testing workload	0	2	10	7	2	0,61	0,20
c) The insufficient awareness of the different assessment methods	0	2	10	9	0	0,58	0,16
d) The large number of students in the class	1	3	3	12	2	0,63	0,26
e) The insufficient teaching time	1	0	5	9	6	0,73	0,25
f) Students' low achievement level	0	2	6	11	2	0,65	0,20

What comes out by our first results (Table 3) is that the teachers attribute the influence on choosing assessment techniques mostly on the curriculum workload and the insufficient teaching time, whereas they eliminate the effect of personal factors, such as their insufficiency in using different assessment techniques.

Regarding their beliefs about their skills in using techniques of formative assessment (Table 4), teachers appear to feel more confident in using individual activities for assessing students' or oral questioning, classroom discussion and classroom observation. In contrast to these more commonly used methods of assessment, using students' self-assessment and peer-assessment and individual interviews with students appear to create insecurities to teachers, maybe because of their less frequent use in the classroom.

Table 4

Frequencies, mean and standard deviation of teachers' beliefs about their skills in using techniques of formative assessment

	No Answer	Not skilled	Less skilled	Skilled	Totally skilled	MEAN	SD
How skilled do you think you are in applying the following assessment techniques?							
a) Classroom discussion	0	0	2	14	5	0,79	0,14
b) Classroom observation	1	0	0	14	6	0,79	0,21
c) Individual interviews with students	0	1	8	8	4	0,68	0,21
d) Assessing students' individual activities	0	0	1	13	7	0,82	0,14
e) Assessing students' group activities	0	0	3	13	5	0,77	0,16
f) Oral questioning	0	0	0	15	6	0,82	0,12
g) Assessing students' presentation skills	0	0	3	14	4	0,76	0,15
h) Students' self-assessment	0	1	7	12	1	0,65	0,17
i) Students' peer-assessment	0	3	11	6	1	0,56	0,19

5 THE EFFECTIVE USE OF FORMATIVE ASSESSMENT RESULTS

5.1 THE USE OF FEEDBACK

Feedback is an important dimension of formative assessment, either as provided by teachers to students through questions, comments etc., by students to the teacher, or by students between them or, in relation to self-assessment and peer-assessment practices. Thus, the use of feedback as a result of formative assessment is included in our third research question about the teachers' beliefs about the way the results of formative assessment can be used effectively.

The power of feedback becomes evident in different definitions of formative assessment that highlight the importance of integrating feedback in instruction. For example, Sadler (1998) refers to formative assessment as specifically intending to provide feedback on students' performance for improving and accelerating their learning. In line with this opinion, Cauley and McMillan (2010) explain that by showing the students specific misunderstandings or errors that frequently occur in a content area or a skill set, and showing them how they can adjust their approach to the task, students can see what they need to do to maximize their performance. As a result, feedback to students that focuses on developing skills, understanding, and mastery, and treats mistakes as opportunities to learn is particularly effective for their progress in learning and gives students hope and positive expectations for themselves. Besides the focus on the positive effects of providing feedback to students, researchers emphasize also on gaining feedback from students about their learning and understanding. Actually, Hattie (2009) adds that a powerful influence of formative assessment on achievement is the meaningful feedback from students as to what they know and where they make errors or have misconceptions.

Despite the fact that providing feedback to students occurred as one of the formative assessment techniques in the previous session, in this section we focus on feedback in the sense of incorporating the information and results of feedback for improving the students' teaching and learning. Therefore in this section we discuss about how feedback should be provided, referring to how and when it should be provided and what the results of providing effective feedback to students are. Nicol and Macfarlane-Dick (2004), suggest that the good feedback practice facilitates the development of self-assessment (reflection) in learning, encourages teacher and peer dialogue around learning, helps clarify what good performance is (goals, criteria, expected standards), provides opportunities to close the gap between current and desired performance, delivers high quality information to students about their learning, encourages positive motivational beliefs and self-esteem and provides information to teachers that can be used to help shape the teaching. In order to be able to benefit in the aforementioned ways, these researchers provide also suggestions drawn from research about particular strategies that increase the quality of feedback, in relation to the way and the time feedback should be provided. Specifically, these strategies include making sure that feedback is provided in relation to pre-defined criteria (paying attention to the number of criteria) providing feedback soon after a submission, providing corrective advice not just information on strengths/weaknesses, limiting the amount of feedback so that it is used, prioritizing areas for improvement and focusing on students with greatest difficulties.

Sadler (1998) raises an important issued regarding the use of feedback, turning the focus on the way the students can reclaim and benefit from feedback. He notes that for feedback to act, the teacher has to provide a verbal statement about the quality of the students' work (the reasons for the judgment and ways in which some of the shortcomings could be remedied). Therefore, students should also be trained in how to interpret feedback, how to make connections between the feedback and the characteristics of the work they produce, and how they can improve their work in the future. Sadler (1998) successfully claims that we cannot simply assume that when students are given feedback they will know what to do with it. This is indeed an important factor to take into account when providing feedback to students, in order not only to provide comments about their performance or errors, but also to include particular suggestions and solution about ways that can help the students overcome their weaknesses and improve themselves.

Based on the literature regarding the role of feedback in formative assessment, we extracted some opinions which were then turned to statements, for examining the teachers' beliefs regarding how and when feedback should be provided and what the benefits of giving feedback on the students' learning and the teachers' teaching practices are. Table 5 includes the results about the statements reflecting the teachers' beliefs about feedback.

Table 5

Frequencies, mean and standard deviation of teachers' beliefs about feedback

	No Answer	Strongly Disagree	Rather Disagree	Rather Agree	Strongly Agree	Mean	SD
Providing feedback to a student can be achieved by: a) providing a verbal statement about the quality of work itself (the reasons for the judgment and ways in which some of the shortcomings could be remedied).	1	0	5	9	6	0,73	0,25
b) showing students' specific misunderstandings or errors that frequently occur in a particular mathematical content area or a skill set.	0	0	2	7	12	0,87	0,17
c) showing students how they can adjust their approach to the task.	0	0	2	10	9	0,83	0,16
The results' of formative assessment should be: a) announced to the whole class.	0	1	13	5	2	0,60	0,19
b) discussed between parents and teacher.	0	0	6	10	5	0,74	0,19
c) discussed between the pupil and the teacher.	0	0	1	12	8	0,83	0,14
Formative assessment works best when the teacher avoids grading practices and comments that show students how their performance compares to other students.	0	0	9	8	4	0,69	0,19
The quality of feedback increases when providing feedback right after a submission.	0	0	2	14	5	0,79	0,14

Feedback about the students' progress in learning mathematics gives hope and positive expectations for themselves.	0	0	3	14	4	0,76	0,15
Formative assessment during instruction provides feedback that help students correct their errors.	0	0	0	17	4	0,80	0,10
Formative assessment during instruction helps the teachers identify and implement instructional correctives.	0	0	1	14	6	0,81	0,13

Our first results indicate that the teachers recognize the benefits of feedback on their practices and the students' cognitive and affective domain. In particular, the teachers mostly agree with using the students' errors for giving feedback and helping them overcoming them and have positive beliefs about the positive results of feedback on increasing the students' motivation. Furthermore, they consider more important discussing the results of assessment with the students and then involve also their parents in the feedback process. However, the teachers prefer individualized feedback, as they are negative in announcing assessment results to the whole class. They also prefer avoiding grading practices and comments that show students how their performance compares to other students and using practices and comments that leads the students comparing their performance with each other. Regarding the time of providing feedback, the teachers strongly believe that feedback must be provided very close to the students' work.

5.2 TEACHERS' BELIEFS TOWARDS MATHEMATICAL ERRORS

As shown by our first results, the use of students' errors is an important dimension of formative assessment, as it helps the teachers modify their practices for helping the students correcting them, but also the students in identifying their weaknesses and try overcoming them. We strongly agree with this, thus in this section the important of using and interpreting the students' errors is going to be discussed emphasizing on the role of the teachers' beliefs about errors, which is a part of our objectives for our project when examining the teachers' beliefs for formative assessment. Wragg (2001) supports that "if students are to learn from their assessment, then correction of errors and discussion of what they have done is essential" (p.74). This strengtheners our opinion about the significance of studying teachers' beliefs regarding the origination of the students' errors, as these beliefs can affect the way the teachers will decide to discuss about errors and work with them for helping the students overcoming them.

In fact, the identification of mistakes helps teachers decide how to identify and meet pupils' learning needs and how to use their teaching time and their resources (Kyriakides, 1999). The reason on which the teachers attribute the errors will affect their decisions for their future intervention teaching practices. Therefore, the students' errors can have a formative use, as the teachers can exploit this information for modifying their future actions (Gagatsis & Kyriakides, 2000). Thus, decisions about the next learning steps follow from the formative identification of pupils' errors (Desforges, 1989). And this is particularly important, because a teaching plan which is organized in such a way, might help teachers to plan class and individual programs of work according to the different performance levels of the pupils (Gagatsis & Kyriakides, 2000).

Therefore, in order to be able to provide suggestions through our project towards the effective formative use of students' errors we have to get an insight to the teachers' beliefs about the source of these errors. By encouraging the analysis of pupils' errors in the training model we plan to develop, we can enable teachers to seek specific information about individual pupils' thinking and understanding and then adjust the level of content to match individual pupils' performance levels. A number of studies (Milhaud, 1980; Charnay, 1989; Economou, 1995) revealed that teachers attributed errors mainly to the pupils' lack of interest or lack of preparation. Gagatsis and Christou (1997) examined also the extent to which the didactical and epistemological approaches to the concept of error influence teachers' attitudes investigated. They actually examined the interpretations that primary school teachers give about their pupils' errors. The results of their study showed that the majority of teachers hold similar beliefs. For example, 90% of primary school teachers attributed errors to the psychological situation of the pupil, 80% of the teachers attributed errors to the limited capabilities of the pupil, and 85% considered the lack of knowledge as a reason for errors. Gagatsis and Kyriakides (2000), examined not only whether teachers agreed with aspects of the didactical and epistemological approach to the concept of error but also whether they could identify errors of their pupils associated with the concepts of obstacle and didactic contract. In their study, teachers' responses revealed that items concerned with reasons for errors can be classified into four broad categories. These are pupils' characteristics, teachers' role, the mathematical knowledge, and the rules which pupils are supposed to follow in a typical mathematics classroom. The survey, also, showed that teachers supported that errors in mathematics are often associated with the characteristics of the pupils. This seems to be in line with the findings of a number of studies (i.e Charnay, 1989; Economou, 1995; Milhaud, 1980) which revealed that teachers attributed errors mainly to the pupils' lack of interest or lack of preparation. Finally, the teachers considered error analysis as a significant way of improving their teaching practice.

For examining the teachers' beliefs for errors we used some of the statements used in the questionnaire of Gagatsis and Kyriakides (2000). In particular, our questionnaire included 12 statements expressing beliefs that corresponded to Gagatsis and Kyriakides' classification of errors in four categories (see table 6).

The results of the studies mentioned before reveal that the teachers mainly attribute errors to factors related to the students' characteristics and knowledge. Our first results (Table 6) are in line with these results, as the particular teachers' beliefs about errors because of the students' characteristics and knowledge have the higher intensity in relation to the rest factors. The results also show that the teachers consider that their role in teaching has the less effect on the students' errors.

Table 6

Means and standard deviation of the beliefs about each factor related to errors

STATEMENTS	MEAN	SD
Pupils		
R10: Errors are associated with the way the student studies and prepares himself/herself.	0,60	0,20
R11: Errors are associated with student's attitude towards mathematics	0,76	0,22
R12: Errors are associated with the psychological situation of the student.	0,71	0,21
R14: Errors are due to the limited capabilities of students.	0,57	0,23
Total	**0,72**	**0,18**
Teachers		
R9: Errors are associated with the text of the problem.	0,81	0,22
R13: Errors are associated with inappropriate ways of teaching.	0,68	0,20
R19: Errors are due to the fact that an inappropriate question for the ability of the student is given.	0,69	0,19
Total	**0,60**	**0,18**
Characteristics		
R8: Errors are associated with lack of knowledge.	0,65	0,22
R15: Errors are due to wrong or incomplete knowledge about a concept taught previously.	0,56	0,19
R16: Errors are due to previous correct knowledge which is not appropriate in a new situation.	0,58	0,21
R17: Errors are due to a confusion of the model needed for completing a task with an already known model.	0,61	0,26
Total	**0,68**	**0,18**
Rules		
R18: Errors are due to the students' tendency to fulfill their teacher's wishes without examining them.	**0,74**	**0,24**

For further analyzing the relations between these different beliefs with other statements expressing beliefs about the use of results of formative assessment, the implicative statistical analysis (Gras, Régnier, Marinica & Guillet, 2013) was performed using the software CHIC (Classification Hiérarchique, Implicative et Cohésitive) (Bodin, Coutourier, & Gras, 2000). These methods of analysis determine the implicative relations of the variables (Gras, Suzuki, Guillet &

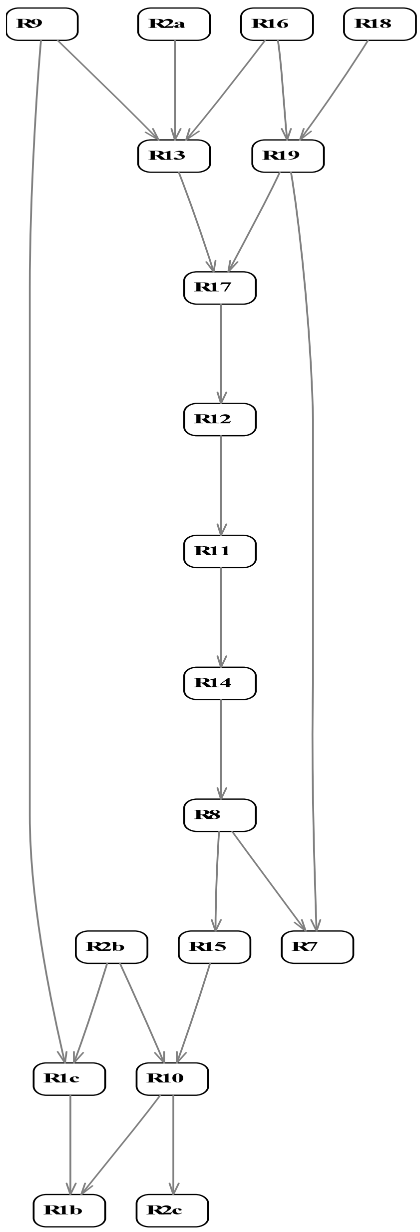

Figure 1. Implicative diagram

Spagnolo, 2008). This analysis actually aims at giving a statistical meaning to expressions like: "if we observe variable A in a subject, then in general we observe variable B in the same subject". Thus, the underlying principle of the implicative analysis is based on the quasi-implication: "if A is true, then B is more or less true". An implicative diagram represents graphically the network of the quasi-implicative relations among the variables of the set V. In this study the implicative diagrams contain implicative relations, indicating whether the existence of a particular belief implies an effect on the creation of belief.

The implicative diagram (Figure 1) reveals relations between the teachers' beliefs about the source of errors, the formative use of errors and the about how the results of formative assessment should be used. In specific, an implicative chain is formed between the statements expressing the teachers' beliefs about errors. At the top of this chain there are the beliefs regarding the errors associated with the teacher and some of the beliefs relating errors with the characteristics of the concept. This shows that the teachers' beliefs about errors as a result of their practices or some characteristics of the concept have a greater influence on their systems of beliefs about errors, than their beliefs about errors related to characteristics of the students or other characteristics of the concept. In fact this is indicated by the lower position of these statements in the implicative chain.

Also, at the lower part of this diagram, a group of variables appears, relating the beliefs about errors with particular ways of giving feedback and about the use of the results of formative assessment. In this group of beliefs the teachers put in the centre of formative assessment process the pupils, by expressing how the pupils should get informed about their errors and how to use in an effective way this information for correcting them. In particular, the discussion about the results' of formative assessment between parents and teachers appears to be important for the teachers (R2b), when they believe that that errors are associated with the way the student studies and prepares himself/herself (R10) and that providing feedback to a student can be achieved by showing students how they can adjust their approach to the task (R1c). The attribution of errors to the way the student studies (R10) also leads teachers in considering that the results' of formative assessment should then be discussed between the pupil and the teacher (R2c) and that providing feedback to a student can be achieved by showing students' specific misunderstandings or errors that frequently occur in a particular mathematical content area or a skill set (R1b). Another relation that should be mentioned is that if the teachers believe that errors are due to the fact that an inappropriate question for the ability of the student is given (R19) they also believe that formative assessment during instruction helps the teachers identify and implement instructional correctives (R7). This relation reveals the importance of the teachers' beliefs about errors influence their beliefs about the use of formative assessment.

The relations discussed above indicate the importance of studying the teachers' beliefs about the source of errors, as they appear to be influencing the way of providing feedback and how they can use effectively the information they get when using formative assessment for helping their students overcoming their difficulties. Thus, the teachers' beliefs seem to have an effect on the formative use of errors in the teaching and learning of mathematics.

6 TEACHERS' TRAINING

Besides the teachers' beliefs about the role, the characteristics, the techniques and the use of results of formative assessment, we were interested in examining the teachers' needs for developing their skills in the different dimensions of formative assessment. Collecting information regarding the teachers' needs in crucial for developing our training model and our repository. Aligning the development of our training methodology with the teachers' needs will increase the suitability and the usability of our model. For meeting this goal, the last part of our questionnaire included statements for examining the teachers' needs for further training on methods and practices regarding assessment. Table 7 presents the first results about the teachers' training on these different topics.

Table 7

Frequencies of teachers' preferences for training on assessment topics

Statements	**F (N=21)**
Given assessment workshops in the future, please indicate which topic(s) you would like to attend.	
Methods to assess students' achievement.	14
Using assessment methods to develop teachers' abilities to teach effectively.	13
The application of different assessment methods.	12
Analyzing assessment method results.	11
Using assessment methods to provide students with feedback.	11
Using assessment methods to improve students' abilities.	11
Students' self-assessment.	9
Higher order questioning techniques.	8
Use of misconceptions.	8
Peer assessment.	8
Encourage students' participation in classroom activities.	7
Feedback as comments and not grades.	5
Oral feedback.	3
Sharing assessment criteria.	3
I would not like to attend any assessment workshop.	0

The teachers express their desire to get further training mostly in methods to assess students' achievement and to develop their own abilities in teaching effectively, but also for applying different assessment methods. They are also interested in improving their skills in analyzing the results of assessment and in using assessment methods to provide feedback to students and improving the students' abilities. Providing oral feedback or feedback as comments and not grades and sharing assessment criteria with the students are the least preferable topics on which the teachers would like to be trained. It is encouraging than none of the 21 participants express that he/she would not like to attend any training on assessment.

7 THE ADOPTED DEFINITION OF THE PROJECT

Based on the results of the literature review, a synthesis of different definitions was done in order to be able to express the way formative assessment in mathematics teaching and learning is defined in our project. Therefore, according to our synthesis, we resulted in providing the following extended definition and description of formative assessment.

"Formative assessment is connected with a concept of learning, according to which all students are able to acquire, at an adequate level, the basic skills of a discipline. The learning passes through the use of teaching methodologies which can respond effectively to different learning times for each student, their different learning styles, and their zones of proximal development. Formative assessment is an assessment FOR teaching and learning. It is part of the teaching-learning process and regulates it. It identifies, in an analytical way, the strengths and weaknesses of student's learning, in order to allow teachers to reflect on and modify their own practices. It allows, in a form of formative feedback, to establish a dialogue between teacher and student and to design educational interventions; It also promotes and fosters the learning of all students through differentiated teaching that ensures each student different rhythms and different teaching and learning strategies, involving at the same time the student in the analysis of own errors/weaknesses and own ability to promote self-assessment and peer-assessment and active participation in the teaching-learning process.

It is intended to give information, feedback and feed forward – in and outside of the classroom – related to the development of mathematical life-skills. In particular, it involves the different components of mathematical learning of the students (conceptual, procedural, semiotic, communicative, problem posing and solving aspects, misconceptions, organization of mathematical experience), the students' beliefs, the students' image of mathematics and of specific segments of mathematics, their behavior and classroom interaction when involved in different mathematical tasks and the outputs of teacher's choices (transposition of mathematical contents, interface between contents and methods)".

Trying to provide a complete and thorough description of formative assessment, we tried to include main points describing the purpose, the techniques and the results of formative assessment, preserving the relation with the literature review and the main axes of our research. The following table (Table 8) is an effort to deconstruct our definition in relation to our main research axes, for making their correspondence more explicit.

Table 8

The definition of FAMT&L project in relation to the research axis

Purpose	**Techniques**	**Results**
	Teachers	
• assessment FOR teaching and learning • regulates teaching-learning process • establish a dialogue between teacher and student	• teaching methodologies which can respond effectively o to different learning times for each student o their different learning styles o their zones of proximal development • formative o feedback o feed forward	• allows teachers to reflect on and modify their own practices. • design educational interventions • the outputs of teacher's choices (transposition of mathematical contents, interface between contents and methods)".

Students		
• students to acquire the basic skills of a discipline • identify the strengths and weaknesses of student's learning • to give information, feedback and feed forward – in and outside of the classroom – related to the development of mathematical life-skills	• promotes students' ability for self-assessment and peer-assessment	• learning for all students through differentiated teaching (different rhythms and different teaching and learning strategies) • students' active participation in the teaching-learning process. • involving the student in the analysis of own errors/ weaknesses

Particular elements of our definition are categorized in relation to the main axes regarding the purpose, the techniques and the results of formative assessment. We intentionally do not include the dimension of the teachers' training for formative assessment in our definition, as it is a dimension that has an indirect relation to the application of formative assessment in the classroom.

8 THE STRUCTURAL ORGANIZATION OF FORMATIVE ASSESSMENT IN MATHEMATICS

8.1 STRUCTURAL EQUATION MODELING AND CFA

Confirmatory factor analysis (CFA), by using the EQS program, will be used to explore the structural organization of the various dimensions of formative assessment in mathematics (Bentler, 1995). Structural equation modeling (SEM) is a statistical methodology that takes a hypothesis testing (i.e. confirmatory) approach to the multivariate analysis of a structural theory bearing on some phenomenon (Byrne, 1994). This theory concerns causal relations among multiple variables (Bentler, 1988). These relations are represented by structural, namely regression equations, which can be modeled in a pictorial way to allow a better conceptualization of the theory involved.

CFA is used in situations where the researcher aims to test statistically whether a hypothesized linkage pattern between the observed variables and their underlying factors exists. This a priori hypothesis draws on knowledge of related theory and past empirical work in the area of the study. In this case the knowledge comes from the synthesis on the various dimension of formative assessment based on the results of the literature review.

CFA allows the researcher to test the hypothesis that a relationship between the observed variables and their underlying latent construct(s) exists. The researcher uses knowledge of the theory, empirical research, or both, postulates the relationship pattern a priori and then tests the hypothesis statistically. Traditional statistical methods normally utilize one statistical test to determine the significance of the analysis. However, Structural Equation Modeling, CFA specifically, relies on several statistical tests to determine the adequacy of model fit to the data. The chi-square test indicates the extent of difference between expected and observed covariance matrices. A chi-square value close to zero indicates little difference between the expected and observed covariance matrices. In addition, the probability level must be greater than 0.05 when chi-square is close to zero (Suhr, 2006).

The basic steps that a researcher follows in carrying out CFA are described below: The model is specified, based on knowledge of relevant theory and previous empirical research. Using a model-fitting program, such as EQS, the model is analyzed so that the estimates of the model's parameters with the data are derived. Then the tenability of the model is tested based on data that involve all the observed variables of the model (Byrne, 1994; Kline, 1998). The number of levels that the latent factors are away from the observed variables determines whether a factor model is called a first-order, a second-order or a higher order model. Correspondingly, factors one level removed from the observed variables are labeled first-order factors while higher-order factors which are hypothesized to account for the variance and co-variance related to the first-order factors are termed second-order factors. A second or a higher order factor does not have its own set of measured variables. In this study a third-order model will be considered.

A structural equation model involves two basic types of components: the variables and the processes or relations among the variables. A schematic representation of a model, which is termed path diagram, provides a visual interpretation of the relations that are hypothesized to hold among the variables under study. The proposed model of the study is presented in Figure 2. The design of the proposed model occurred in relation to the main axes of investigation after the finalization of the questionnaire.

8.2 THE PROPOSED MODEL

The model is expected to be a third-order model consisting of six first-order factors, four second-order factors and one third-order factor. The four second-order factors will represent the purpose of formative assessment, the techniques used during formative assessment, the use of results of formative assessment and teachers' training in formative assessment. These four second-order factors are anticipated to be regressed on a third-order factor standing for the formative assessment in mathematics.

More specifically, on the second-order factor that stands for the purpose of formative assessment the first-order factors about the role of formative assessment (F1) and the benefits of formative assessment (F2) are anticipated to be respectively regressed. The next second-order factor that corresponds to the techniques used in formative assessment is expected to be related to two first-order factors reflecting the formative assessment practices (F3) and the factors that determine the choice of the different practices (F4). The second-order factor about the results of formative assessment will consist of two first-order factors, formed by questions about the use of formative feedback (F5) and the use of the students' mathematical errors (F6) respectively. Finally, the second-order factor corresponding to the teachers' training is anticipated to be formed by the questions examining the teachers' opinions and needs about their training in assessment techniques.

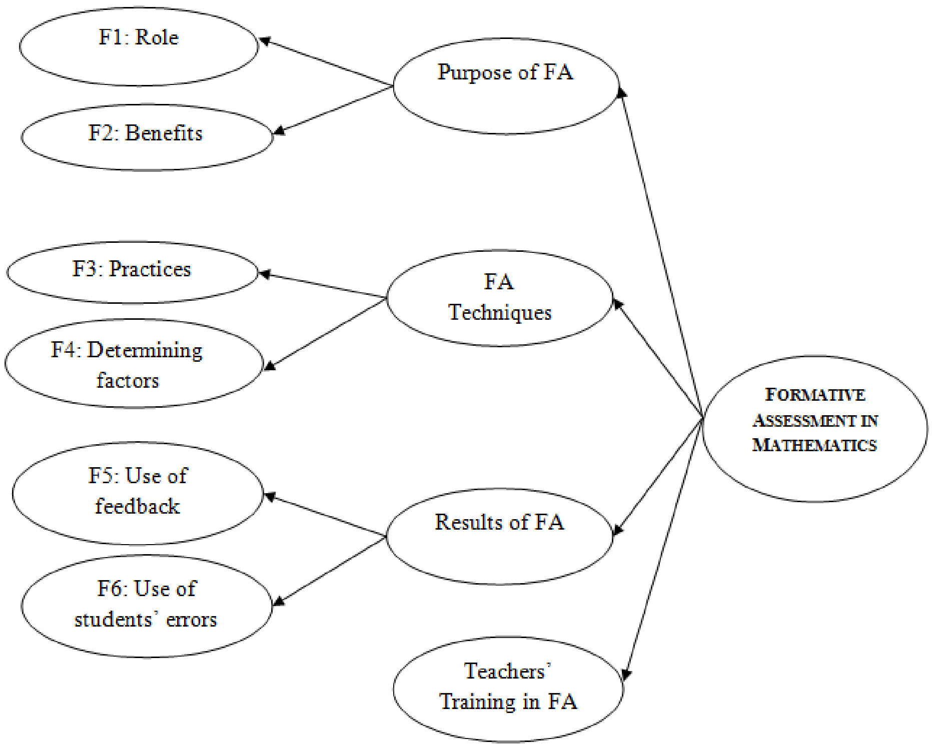

Figure 2. The proposed structure of teachers' beliefs for formative assessment

9 DISCUSSION

Despite the fact that much has been written about the purposes of assessment, research about the teachers' beliefs about the purpose of assessment and the use of the information collected during the assessment process is still rather limited. In this paper an effort to study the teachers' beliefs about formative assessment in mathematics is described, within the actions of a research project about formative assessment in mathematics teaching and learning. The study of teachers' beliefs has an important contribution in efforts about designing and implementing teaching and learning practices. This is evident from research results, which show that the teachers' teaching practice and implementing changes in the classroom are affected by their beliefs about their subjects and approaches to teaching (Thompson, 1992). Research indicates also that, regarding the teaching of mathematics, the teachers' practices are shaped by their beliefs about mathematics and the nature of teaching and learning (Fernandez, 1997).

Consequently, the results of our study can provide us information about what the teachers think and believe about formative assessment in mathematics. The first results from the pilot administration of our questionnaire provide some first indications about the teachers' beliefs regarding the different aspects of formative assessment. We shall clarify that we do not consider that our results can be generalized, as they are results from the pilot phase of this survey and the sample size is very small.

Furthermore, the verification of the proposed three-level hierarchy will provide a thorough description of the role and use of formative assessment in mathematics, based on the spectrum of the included dimensions. We strongly believe that the structure we propose can be verified, because all the elements we used for designing and conducting our research are extracted from the literature in assessment.

Our proposed model can be also related to assessment process as proposed by NCTM (1995). According to the NCTM standards, an assessment process is provided thought four interrelated phases, not necessarily linear, that highlight principal points at which critical decisions need to be made (Figure 3).

Figure 3. Assessment process according to NCTM (1995)

Within each phase of the assessment process decisions and actions must be defined. Therefore, for each phase particular questions are posed. Regarding the phase of planning the assessment, some of the questions are about what purpose does the assessment serve, what framework is used to give focus and balance to the activities, what methods are used for gathering and interpreting evidence? In relation to our model, these questions show that the phase of planning seems to correspond mainly to the factor of purpose, but also to the factor of techniques of assessment. The second phase, which is for gathering evidence the questions are about the way the activities and tasks of assessment can be created or selected, the procedures selected for engaging students in the activities and about how the methods for creating and preserving evidence of the performances can be judged. In our model, the phase of gathering could be related to the factors regarding the techniques of assessment and what are for factors that influence the selection and application of these techniques.

Interpreting the evidence constitutes the third phase of the NCTM model. In these phase the decision to be taken base on questions about how can the quality of the evidence be determined, how can an understanding of the performances to be inferred from the evidence and how will the judgments be summarized as results? Finally the last phase of using the results relates with questions about the way the results can be reported, how should inferences from the results be made, what action will be taken based on the inferences and about how it can be ensured that these results will be incorporated in subsequent instruction and assessment. Based on the main points of these two last phases, we find a correspondence with the factor of results in our proposed model which relates to the way the results of formative assessment can be used in an effective way for improving teaching and learning of mathematics.

Concluding, teachers' beliefs, as reflected in their practice, influence students' beliefs (Franke, Fennema, & Carpenter, 1997). Consequently, teachers have a remarkable influence on students' construction of their beliefs through the ways in which they present the subject matter, the kinds of task they set, assessment methods, procedures and criteria (Pehkonen, 1998). Thereafter, gaining access to the teachers' beliefs will give us the opportunity to design relevant teaching material, based on their needs, in order to have the chance to achieve a change in their classroom practices towards not only the effective implementation of formative assessment practices during their teaching, but also towards the construction of their students' positive beliefs about assessment. As the relationship between teachers' beliefs and classroom practice is dynamic with each influencing the other (Fernandez, 1997), we believe that we will be able to bring a change to the teachers' beliefs about the use of formative assessment, with this change to be reflected in the effective use of formative assessment practices.

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